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MODELS OF ECONOMIC EQUILIBRIUM: COMPARATIVE ANALYSIS AND SEARCH FOR BALANCE

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This work presents models of economic equilibrium by L. Walras and V. Kardash. The Walras model, created in the second half of the XIX century, is the first mathematical model of general economic equilibrium. It is based on the concept of perfect competition and assumes that all market participants act rationally, striving to maximize their benefit. The model defines an abstract system in which rational individuals operate, optimizing their objective functions. At the same time, there is perfect competition, which means that no individual participant can influence the market situation. The Kardash's model, developed in the XXI century, is a more modern and realistic model. It is based on the concept of a compromise economic balance and takes into account the presence of conflicts of interests between market participants. The article highlights the main differences between the models of L. Walras and V. Kardash, such as: the concept of modeling economic equilibrium; the mechanism for achieving economic equilibrium; the process of formation of equilibrium prices; supply and demand functions; determination of the state of equilibrium; degree of aggregation. Analyzing these differences, we come to the conclusion that V. Kardash's model is a more modern and promising model for modeling equilibrium states under the conditions of a market economy. The model has such advantages as: constructability; practical orientation; socio-economic orientation. Kardash's model uses specific marginal functions of demand and supply, which allows it to be applied to practical problems. It takes into account the conflict-compromise nature of market relations, which makes it more realistic and allows modeling not only competitive, but also socio-economic equilibrium. The article establishes that V. Kardash's model is a qualitatively new stage in the development of the theory of general economic equilibrium. It allows more complete and accurate reflection of real economic processes.

Key words: *economic equilibrium, L. Walras model, V. Kardash model, competitive mechanism, compromise equilibrium.*

Білоусова Т. П. Моделі економічної рівноваги: порівняльний аналіз та пошук балансу
У цій роботі представлені моделі економічної рівноваги Л. Вальраса та В. Кардаша. Модель Вальраса, створена у другій половині XIX століття, є першою математичною моделлю загальної економічної рівноваги. Вона основана на концепції досконалої конкуренції та передбачає, що всі учасники ринку діють раціонально, прагнучи максимізації своєї вигоди. Модель визначає абстрактну систему, у якій діють раціональні індивіди, які оптимізують свої цільові функції. При цьому має місце досконала конкуренція, яка

означає, що жодний окремий учасник не може вплинути на ринкову ситуацію. Модель Кардаша, розроблена в XXI столітті, є більш сучасною та реалістичною моделлю. Вона грунтується на концепції компромісної економічної рівноваги та враховує наявність конфліктів інтересів між учасниками ринку. В статті виділяються основні відмінності між моделями Л. Вальраса та В. Кардаша такі, як: концепція моделювання економічної рівноваги; механізм досягнення економічної рівноваги; процес формування рівноважних цін; функції попиту та пропозиції; визначення стану рівноваги; ступінь агрегування. Аналізуючи ці відмінності, приходимо до висновку, що модель Кардаша є більш сучасною та перспективною моделлю для моделювання рівноважних станів за умов ринкової економіки. Модель має такі переваги, як: конструктивність; практична спрямованість; соціально-економічна спрямованість. В моделі Кардаша використовуються конкретні граничні функції попиту та пропозиції, що дозволяє застосувати її до практичних завдань. Вона враховує конфліктно-компромісний характер ринкових відносин, що робить її більш реалістичною та дозволяє моделювати не лише конкурентну, а й соціально-економічну рівновагу. В статті встановлено, що модель В. Кардаша є якісно новим етапом у розвитку теорії загальної економічної рівноваги. Вона дозволяє більш повно та точно відображати реальні економічні процеси.

Ключові слова: *економічна рівновага, модель Л. Вальраса, модель В. Кардаша, конкурентний механізм, компромісна рівновага.*

Introduction. The theory of general economic equilibrium examines issues of interaction between the main participants in the economy – producers and consumers – in terms of prices and volumes of factors and goods. In general terms, economic equilibrium involves the establishment of prices at which all participants in the economic system achieve the best results for themselves. The state of economic equilibrium is distinguished by the fact that none of the participants is interested in changing it with the help of the means at its disposal.

The first mathematical model of general economic equilibrium was created in the second half of the 19th century by the outstanding French economist L. Walras [1]. Despite the fact that this model was purely theoretical, the ideas contained in it had a significant impact on the mathematical modeling of economic processes. Assessing Walras's contribution to economic science, the historian of economic thought J. Schumpeter argued that «We owe to Walras the concept of an economic system and a theoretical apparatus that, for the first time in the history of our economy, effectively captured the pure logic of the relationships and interdependencies of quantitative economic indicators». Such prominent economists as A. Wald, G. Cassel, K. Arrow, J. Debreu, J. von Neumann, L. McKenzie, J. Hicks, P. Samuelson, V. Pareto and others worked in the field of economic equilibrium modeling [2-3].

Formulation of the problem. Currently, the concept of compromise economic equilibrium is becoming increasingly developed and widespread, in line with which lies the compromise economic analysis first proposed by V. Kardash [4]. A distinctive feature of compromise analysis is that it is based on indicators expressing the state of compromise alignment of interests of economic entities. In this regard, of particular interest is a comparative analysis of the historically first model of economic equilibrium of L. Walras with one of the modern models of economic equilibrium developed by V. Kardash.

Presenting main material. *1. Formal and substantive analysis of the models of L. Walras and V. Kardash.* Walrasian economics is an abstract system in which rational individuals act, optimizing their objective functions. All necessary information is contained in prices and is equally available to all participants in the economic system. In this case, perfect competition takes place, meaning that no individual participant can influence the market situation, and it, in turn, provides them with equal opportunities to realize their preferences. The main idea of the Walras model is that,

under a certain price system, all participants in the economic system achieve the best result for themselves, and a distribution of goods and factors occurs that resolves all conflict situations between the participants. This situation in an economic system is called competitive equilibrium.

The formal representation of the Walras model is as follows [1]. An economy with l consumers, m producers and n types of goods is considered. Each consumer is characterized by an income function $K_i(p)$ and a demand function $\Phi_i(p)$. The amount of income of each consumer consists of two values: from the sale of his initial stock of goods b_i bi and income as a result of the consumer's participation in the production of $I_i(p)$. Then we get

$$K_i(p) = \langle b_i, p \rangle + I_i(p), \quad i = 1,2,\ldots,l.$$

The consumer demand function has the form

$$\Phi_i(p) = \begin{cases} x_i : x_i \in X_i(p), u_i(x) = \max\limits_{x_i' \in X_i(p)} u_i(x_i'), i = 1,2,\ldots,l; \\ 0, \text{otherwise.} \end{cases}$$

The manufacturer is characterized by a set of production plans $Y_k \in R^n$ and a supply function $\psi_k(p)$ of type

$$\psi_k(p) = \{ y_k \in Y_k : \langle y_k, p \rangle = \max_{y_k' \in Y_k} \langle y_k', p \rangle, k = 1,2,\ldots,m. \}.$$

The set $\left(y_1^*,\ldots,y_m^*,x_1^*,\ldots,x_l^*,p^*\right)$ defines the competitive equilibrium in the Walras model, if

$$y_k^* \in \psi_k(p^*), \quad k = 1,2,\ldots m; \tag{1}$$

$$x_i^* \in \Phi_i(p^*), \quad i = 1,2,\ldots l; \tag{2}$$

$$\sum_{k=1}^{m} y_k^* + \sum_{i=1}^{l} b_i \geq \sum_{i=1}^{l} x_i^*; \tag{3}$$

$$\left\langle p^*, \sum_{k=1}^{m} y_k^* + \sum_{i=1}^{l} b_i \right\rangle = \left\langle p^*, \sum_{i=1}^{l} x_i^* \right\rangle. \tag{4}$$

Conditions (1) and (2) mean that each of the participants, considering prices as given, acts in the best way for himself. Condition (3) means that aggregate demand should not exceed aggregate supply, and condition (4) shows that the cost of goods purchased is equal to the cost of goods sold.

In his model of general economic equilibrium, Walras sought to describe the mechanism of perfect competition, when there are a large number of producers and consumers in the market, so that none of them is able to influence market prices. There is another interpretation of the Walras model, according to which it is considered as a scheme of optimal planning based on the coordination of decentralized decisions. It is assumed that the system is running certain rules for price changes and norms of behavior that ensure the achievement of equilibrium [5-7].

In Kardash's economics, the central position is occupied by the law of economic compromises, the essence of which is that conflicts in economic relations are resolved through compromise coordination of divergent interests [2,4]. For the manufacturer, the parameters that determine its competitiveness and the maximum possible profit from selling goods on the market are important. For the consumer, the interest is focused on

the parameters that determine his ability to pay and the maximum possible satisfaction from his purchase on the market. In the process of forming a market compromise, the interests of the producer and consumer are coordinated, as a result of which a resultant interest is developed, expressed by the parameters of a mutually acceptable transaction (P,Y), where P is the price of the product, Y is the volume of purchase and sale of the product. The main idea of the Kardash model is that by compromising the conflicting interests of economic entities, an equilibrium price system is formed, in which the maximum reserve of economic power of consumers and producers is realized. This market situation is called a compromise equilibrium.

Now let us present a formal representation of the Kardash model. We consider an economy that includes n interacting commodity markets. Let D_j be the maximum possible amount of money available to the buyer in the market for the j th product. Then, at price P_j, the consumer's boundary demand function will have the form $Y_j(P_j) = \frac{D_j}{P_j}, \quad j = 1,2,\ldots,n.$

Let D'_j be the minimum required profit that a manufacturer must receive in the market for the j th product, and c_j be the costs of production and sale of a unit of product. Then the manufacturer's boundary supply function will take the form:

$$Y'_j(P_j) = \frac{D'_j}{P_j - c_j}, \quad j = 1,2,\ldots,n.$$

Let us define the set of permissible transactions M in a system of interconnected commodity markets:

$$M = \cap M_j = \left\{(P,Y):(P_j - c_j)Y_j \geq D'_j; P_j Y_j \leq D_j, j = 1,2,\ldots,n.\right\}.$$

The set $(P_1^*,\ldots,P_n^*,Y_1^*,\ldots,Y_n^*)$ specifies a compromise equilibrium in the Kardash model if

$$(P_1^*,\ldots,P_n^*,Y_1^*,\ldots,Y_n^*) \in M; \tag{5}$$

$$P_j^* = \arg\max_{P_j} \Delta Y_j = \arg\max_{P_j}\left(\frac{D_j}{P_j} - \frac{D'_j}{P_j - c_j}\right) = c_j \frac{1}{1-\sqrt{\phi_j}}, \quad j = 1,2,\ldots,n. \tag{6}$$

$$Y_j^* = \frac{D_j}{P_j^*}, \quad j = 1,2,\ldots,n. \tag{7}$$

Condition (5) means that in the economic system such compromise transactions are made when buyers are solvent and producers are competitive. Compromise mechanism for coordinating interests consumers and producers is implemented in conditions (6), from which the system of compromise-equilibrium prices $(P_1^*,\ldots,P_n^*)$ is determined. Relations (7) allow us to calculate the compromise-equilibrium purchase and sale volumes $(Y_1^*,\ldots,Y_n^*)$. It can be seen from this that the Kardash model (5)–(7) differs from the Walras model (1)–(4) by its pronounced constructiveness in the construction of boundary functions of supply and demand, as well as the implementation of a compromise mechanism for resolving conflict situations in market transactions. The latter gives reason to say that the Kardash model may have practical applications. In Walras's model, the mechanism for resolving conflict situations is not specified, but is presented only at an abstract level. In addition, the main disadvantage of the Walras

model is that the demand and supply functions are difficult to determine in practice, which reveals its theoretical nature.

2. Analysis of equilibrium states in the models of L. Walras and V. Kardash. The presented models of economic equilibrium differ not only in their ideological content and the mathematical apparatus used, but also in the graphical representation of the equilibrium position. In the Walras model, the state of equilibrium is determined in coordinates (p, y) by the point of intersection of the demand and supply curves, the type of which, as already mentioned, cannot be determined. A graphical illustration of this model is shown in Fig. 1. For his model, Walras did not even try to derive mathematically strict conditions for the existence of equilibrium, but limited himself to only demonstrating a possible mechanism of movement towards equilibrium, the so-called "groping" process [3, 5].

Walras suggested that there could be two types of this process. The first is that the movement starts from an arbitrary vector of prices, and the exchange is made at these "wrong prices". In this case, some participants find themselves are winners and others are losers. Thus, the principle of individual maximization is violated, concluded transactions are canceled and new prices are offered, at which transactions are "concluded" at the next stage, etc. This method involves a long process of trial and error, which in principle can reach an equilibrium.

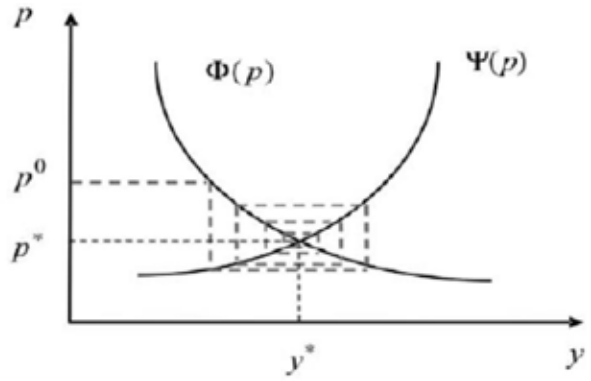

Fig. 1. The "groping" mechanism in the Walras model

Walras considered a process controlled by a certain "auctioneer" to be a more reliable way to achieve equilibrium. The latter first sets arbitrary prices, after which all market participants make conditional transactions and report their results to the "auctioneer". Based on the bids, the "auctioneer" adjusts prices, thus simulating a process of trial and error. Transactions are concluded only after the "auctioneer" announces equilibrium prices. Walras called this process of movement towards equilibrium "groping".

A distinctive feature of the Kardash model is the formation in the same coordinates (P, Y) of a key set of permissible transactions M, limited by specific marginal supply and demand curves. A graphic illustration of the model is shown in Fig. 2.

The equilibrium state corresponds to the point at which the boundary supply and demand curves are as far apart as possible from each other along the Y coordinate. It is at this point that the maximum reserve of economic forces of the consumer and producer is realized, expressed by the ratio $\Delta Y^* = \max \Delta Y(P)$. At the same time, there is clearly no exogenous process of formation of equilibrium prices. What is fundamentally

important here is that the Kardash model contains the idea of market self-regulation of the economy, which distinguishes it from the Walras model, where the regulatory function is carried out by an outside "auctioneer".

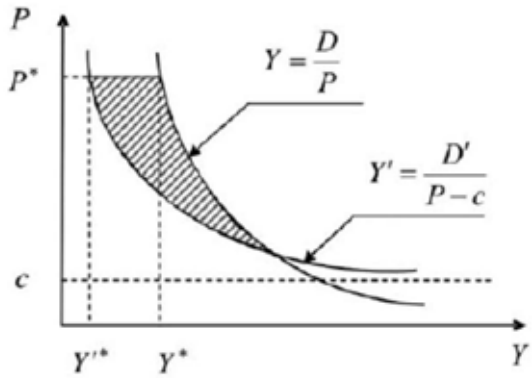

Fig. 2. Set of valid transactions in the Kardash model

Conclusions. Thus, Walras's model, in its purely theoretical formulation, contributed only to the formation of an understanding of general equilibrium and turned out to be poorly suited for modeling and analysis of real economic systems. Walras tried to study the functioning of a perfectly competitive market on an abstract level. At the same time, V. A. Kardash, in his concept of compromise economic equilibrium, managed not only to describe the very real conflict-compromise market mechanism of a competitive economy, but also to give this concept a pronounced constructiveness and practical orientation. The Kardash model is a modern alternative to the Walras model for modeling equilibrium states in a market economy. The concept of compromise economic equilibrium and compromise economic analysis can be considered a qualitatively new stage in the development of the theory of general economic equilibrium. Walras's idea of economic equilibrium as a state in which the maximum of individual utility functions is achieved is far from complete from the point of view of reflecting socio-economic equilibrium. Walras was able to describe only the conflict side of socio-economic interactions based on the action of the competitive mechanism, which revealed the static nature of his approach to modeling economic systems. From the point of view of V. Kardash, the unity and interaction of conflicts and compromises in the system of economic relations represent the internal driving force of the universal mechanism of functioning and self-organization of economic systems.

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